Gravitational Fields Questions

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>Gravitational Constant</td>
<td>$6.67 \times 10^{-11}$ m$^3$kg$^{-1}$s$^{-2}$</td>
</tr>
<tr>
<td>Mass of the Earth</td>
<td>$5.98 \times 10^{24}$ kg</td>
</tr>
<tr>
<td>Mass of the Moon</td>
<td>$7.35 \times 10^{22}$ kg</td>
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<tr>
<td>Radius of the Earth</td>
<td>$6.37 \times 10^6$ m</td>
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<tr>
<td>Radius of the Moon</td>
<td>$1.74 \times 10^6$ m</td>
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<tr>
<td>Mean Earth-Moon distance</td>
<td>$3.84 \times 10^8$ m</td>
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1. What is the size of the force with which the Earth pulls on the moon?

$$F_{gy} = \frac{GMm}{r^2} = 1.99 \times 10^{20} N$$

2. What is the size of the force with which the moon pulls on the Earth?

The answer will be the same as Q1 because gravity works both ways with equal force.

3. What is the gravitational field strength on the surface of the Earth?

$$g = \frac{GM}{r^2} = 9.83 \text{ m/s}^2$$

4. How many times bigger is the gravitational field strength on the surface of the Earth compared to the surface of the moon?

$$g_{moon} = \frac{GM_{moon}}{r_{moon}^2} = 1.62 \text{ m/s}^2$$

$$\frac{g}{g_{moon}} = \frac{9.83}{1.62} = 6.06 \text{ times larger}$$

5. Justify, using the formula for the force due to gravity and Newton’s second law, why all objects (regardless of their mass) accelerate at the same rate towards the surface of a planet.

$$F_g = \frac{GMm}{r^2}$$

$$F = ma$$  \text{ rearranges to give }  \quad a = \frac{F}{m}$$

$$\therefore a = \frac{F_g}{m} = \frac{GMm}{r^2m} = \frac{GM}{r^2}$$

Since the acceleration of the object does not depend on the mass of the object, all objects accelerate at the same rate.
6. What altitude does a spaceship initially on the surface of the Earth need to reach to reduce the force of gravity acting on it by 75%?

\[
F_{\text{surface}} = \frac{GMm}{r_E^2}
\]

\[
F_{\text{alt}} = \frac{GMm}{r_{\text{alt}}^2}
\]

\[
F_{\text{alt}} = 0.25 \times F_g = \frac{0.25GMm}{r_E^2} = \frac{GMm}{r_{\text{alt}}^2}
\]

\[
r_{\text{alt}}^2 = \frac{r_E^2}{0.75} \quad \text{or} \quad r_{\text{alt}} = \frac{r_E}{\sqrt{0.75}} = 1.27 \times 10^7 \text{ m}
\]

\[
\text{altitude above Earth surface} = r_{\text{alt}} - r_E = 6.37 \times 10^6 \text{ m}
\]

7. Planet Hippo has a mass 1.5 times that of Earth but has a radius of only 80% of that compared to Earth. What will the gravitational field strength be at the surface of planet Hippo?

\[
g_E = 9.8 \text{ m/s}^2
\]

\[
g_H = \frac{GM}{r_H^2} = \frac{G(1.5M_E)}{(0.8r_E)^2} = 2.34 \times \frac{GME}{r_E^2} = 2.34 \times g_E
\]

\[
g_H = 2.34 \times 9.8 = 22.9 \text{ m/s}^2
\]

8. The Sun has a mass of \(1.99 \times 10^{30}\ \text{kg}\). The Earth orbits around the sun at a mean distance of \(1.50 \times 10^{11}\ \text{m}\). Calculate the largest and smallest net force acting on the moon due to the gravitational field of the Earth and Sun. Be sure to include the direction of the calculated value.

Mass of moon: \(m\)

Earth-moon distance: \(r_{\text{EM}}\)

Sun-Earth distance: \(r_{SE}\)

Sun-moon distance: \(r_{SM}\)

Max net force occurs when both Sun and Earth are pulling in the same direction (occurs when moon is on the far side of the Earth, away from the Sun).

\[
r_{SM} = r_{SE} + r_{EM} = 1.5038 \times 10^{11} \text{ m}
\]

\[
\Sigma F = F_{\text{Sun}} + F_{\text{Earth}}
\]
Minimum net force occurs when the Earth and Sun pull the moon in opposite directions.

\[ \sum F = \frac{GM_s m}{r_{SM}^2} + \frac{GM_m m}{r_{EM}^2} = 6.30 \times 10^{20} \text{ N towards both Earth and Sun} \]

\[ r_{SM} = r_{SE} - r_{EM} = 1.496 \times 10^{11} \text{ m} \]

\[ \sum F = F_{Sun} - F_{Earth} \]

\[ \sum F = \frac{GM_m m}{r_{SM}^2} - \frac{GM_e m}{r_{EM}^2} = 2.37 \times 10^{20} \text{ N towards the Sun (away from the Earth)} \]

Bonus Question:

Since the Earth is also being pulled by the Sun’s gravitational field and objects have the same acceleration in a gravitational field, regardless of its mass (see question 5), both the Earth and moon are accelerating towards the centre of the solar system. So therefore the moon will not fly away from the Earth.